St. Francis Xavier University

Department of Mathematics, Statistics and Computer Science
presents

Properties of the Commuting Graph of the Symmetric Inverse Semigroup

by

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The commuting graph of a finite non-commutative semigroup S, denoted $\mathcal{G}(S)$, is a simple graph whose vertices are the non-central elements of S and two distinct vertices x, y are adjacent if $xy = yx$. This definition generalizes the corresponding concept of the commuting graph of a non-Abelian group.

Our work looks at the commuting graph of the symmetric inverse semigroup $\mathcal{I}(X)$. For a finite set X, let $\mathcal{I}(X)$ be the semigroup of all partial injective transformations on X under composition. The semigroup is universal for the class of inverse semigroups in the sense that every inverse semigroup can be embedded in $\mathcal{I}(X)$ for some finite set X, analog to the situation of the symmetric groups $\mathrm{Sym}(X)$ in group theory.

In 1989, Burns and Goldsmith classified the maximum order abelian subgroups of $\mathrm{Sym}(X)$. We extend this result to the semigroup $\mathcal{I}(X)$. As a consequence, we obtain a formula for the clique number of the commuting graph of $\mathcal{I}(X)$. We also calculate the diameter of $\mathcal{I}(X)$ when $|X|$ is prime or even, and obtain tight bounds on it in the remaining cases.

Refreshments will be served before the talk in Annex 24A